Nonrandom Mating Systems

I) Mixed Mating Model

- **Mating is at random with probability t and selfing with probability s**
- **Out crossing rate is independent of genotype**
- **Useful for hybrid zone analysis where s = intraspecific mating and t = interspecific mating**

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A) Genotype Frequencies with mixed mating

$$U' = sU + tU^2 + tUV + \frac{1}{4}(sV + tV^2)$$

$$= sU + tU^2 + tUV + \frac{1}{4}sV + \frac{1}{4}tV^2$$

$$= sU + \frac{1}{4}sV + tU^2 + tUV + \frac{1}{4}tV^2$$

$$= s(U + \frac{1}{4}V) + t(U^2 + UV + \frac{1}{4}V^2)$$

$$= s(U + \frac{1}{4}V) + t(U + \frac{1}{2}V)^2$$

$$= s(U + \frac{1}{4}V) + tp^2$$

= proportion of AA produced by selfing plus proportion produced by out crossing.

Similarly:

$$V' = s(\frac{1}{2}V) + t2pq$$ **(note the similarity to pure selfing)**

$$W' = s(W + \frac{1}{4}V) + tq^2$$

B) Allele Frequencies with mixed mating

- **Allele frequencies stay the same over time**

$$p_{t+1} = U' + \frac{1}{4}V'$$

$$= s(U + \frac{1}{4}V) + tp^2 + \frac{1}{2}(\frac{1}{2}sV + t2pq)$$

$$= sU + \frac{1}{4}sV + tp^2 + \frac{1}{4}sV + tpq$$

$$= s(U + \frac{1}{4}V + \frac{1}{4}V) + t(p^2 + pq)$$

$$= s(U + \frac{1}{2}V) + t(p^2 + pq)$$

$$= sp + tp$$

$$= p(s + t) \qquad \& \; s + t = 1.0$$

$$= p$$

C) Equilibrium Genotype Frequencies – although allele frequencies don't change, the genotype frequencies do. Is there a point at which starting from any conditions, the genotype frequencies are stable (i.e., change (Δ) in genotype frequencies = 0.0)?

@ time:

$$V_1 = \frac{1}{2}sV_0 + t2pq$$

@ time:

$$V_2 = \frac{1}{2}sV_1 + t2pq$$

$$= \frac{1}{2}s(\frac{1}{2}sV_0 + t2pq) + t2pq$$

$$= (\frac{1}{2}s)^2 V_0 + \frac{1}{2}s\ 2tpq + t2pq$$

In General:

$$V_t = (\frac{1}{2}s)^t V_0 + 2tpq\left[\sum_{i=0}^{t-1}\left(\frac{1}{2}s\right)^i\right]$$

For Any:

$$\sum_{i=0}^{t-1} X^i = \frac{1-x^n}{1-x}$$

Therefore:

$$V_t = (\frac{1}{2}s)^t V_0 + 2tpq\left[\frac{1-\left(\frac{1}{2}s\right)^t}{1-\frac{1}{2}s}\right]$$

As t => ∞

$$V_E = 2pqt\left(\frac{1}{1-\frac{1}{2}s}\right)$$

$$= 2pq\frac{2(1-s)}{2-s}$$

Similarly:

$$U_E = p^2 + pq\frac{s}{2-s}$$

$$W_E = q^2 + pq\frac{s}{2-s}$$

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D) Mixed mating and full selfing

Remember in full selfing:

$F(AA) = p^2 + pqF$

$F(aa) = q^2 + pqF$

$F(Aa) = 2pq(1 - F)$

Does then:

$$\frac{s}{2-s} = F\,?$$

And then:

$U_E = p^2 + pqF$

$W_E = q^2 + pqF$

$V_E = 2pq(1 - F)$

In other words, does:

$$(1 - F) = 1 - \frac{s}{2-s}\,?$$

Or Does:

$$1 - \frac{s}{2-s} = \frac{2(1-s)}{2-s}\,?$$

$$\frac{2(1-s)}{2-s} = \frac{2-2s}{2-s} = \frac{2-s-s}{2-s} = \frac{2-s}{2-s} - \frac{s}{2-s}$$

$$= 1 - \frac{s}{2-s}$$

This gives us a measure of inbreeding in a population in terms of the degree of selfing in a mixed mating population.

Testing:

if s = 1.0 (t = 0.0)

$$F = \frac{s}{2-s} = 1.0$$

$V_E = 2pq(1 - F) = 0.0$

$U_E = p^2 + pqF = p^2 + pq = p(p + q) = p$

$W_E = q^2 + pqF = pq^2 + pq = q(p + q) = q$

$p + q = 1.0 \therefore U_E + W_E = 1.0 \therefore$ **All homozygotes at equilibrium.**

if s = 0.0 (t = 1.0)

$$F = \frac{s}{2-s} = 0.0$$

$V_E = 2pq$

$U_E = p^2$

$W_E = q^2$

$\therefore$ **All genotypes at HW genotypic proportions at equilibrium.**

if 0.0 < s < 1.0

heterozygotes are maintained relative to degree of inbreeding.

E) Rate of approach to Equilibrium in Mixed Mating Model

Have shown:

$$V_t = (\frac{1}{2}s)^t V_0 + 2tpq\left[\frac{1-\left(\frac{1}{2}s\right)^t}{1-\frac{1}{2}s}\right]$$

Set $d = \frac{1}{2}s$; Keep $F = \frac{s}{2-s}$; and $1 - F = \frac{2(1-s)}{2-s} = \frac{t}{1-\frac{1}{2}s}$

$$V_t = d^t V_0 + 2tpq(1 - d^t)\frac{1}{1-\frac{1}{2}s}$$

$$= d^t V_0 + 2pq(1 - d^t)\frac{t}{1-\frac{1}{2}s}$$

$$= d^t V_0 + 2pq(1 - d^t)(1 - F)$$

$$= d^t V_0 + (2pq - 2pqd^t)(1 - F)$$

$$= d^t V_0 + 2pq(1 - F) - 2pqd^t(1 - F)$$

$$= \underline{2pq(1 - F)} + \underline{[V_0 - 2pq\,(1 - F)]\,d^t}$$

Equilibrium Term — Transient Term

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Similarly:

$$U_t = \underline{p^2 + pqF} - \frac{1}{2}\underline{[V_0 - 2pq\,(1 - F)]\,d^t}$$

$$W_t = \underline{q^2 + pqF} - \frac{1}{2}\underline{[V_0 - 2pq\,(1 - F)]\,d^t}$$

II) Assortative Mating

- **Nonrandom mating based on phenotype. Like phenotypes mate preferentially with like (or dissassortatively with unlike; hybrid zone).**
- **Mating is based on female choice**
- **AA mates with AA or aa mates with aa with probability α. Either mates at random with probability $1 - \alpha = \beta$**
- **Aa (heterozygotes or hybrids) mate at random only.**

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$$U' = \alpha(U + \frac{1}{2}Vp) + \beta p^2$$

$$V' = \frac{1}{2}\alpha V + \beta 2pq$$

$$U' = \underline{\alpha(W + \frac{1}{2}Vp)} + \underline{\beta q^2}$$

Assorative Mating	Random Mating

$$p' = U' + \frac{1}{2}V'$$

$$= \alpha(U + \frac{1}{2}Vp) + \beta p^2 + \frac{1}{2}(\frac{1}{2}\alpha V + \beta 2pq)$$

$$= \alpha(U + \frac{1}{2}Vp) + \beta p^2 + \frac{1}{4}\alpha V + \beta pq$$

$$= \alpha(U + \frac{1}{2}Vp) + \frac{1}{4}\alpha V + \beta p^2 + \beta pq$$

$$= \alpha(U + \frac{1}{2}Vp + \frac{1}{4}V) + \beta(p^2 + pq)$$

$$= \alpha(U + \frac{1}{4}V + \frac{1}{2}Vp) + \beta p$$

Note: $U + \frac{1}{2}V = p$, $U + \frac{1}{4}V + \frac{1}{4}V = p$, $U\ \frac{1}{4}V = p - \frac{1}{4}V$

$$= \alpha(p - \frac{1}{4}V + \frac{1}{2}Vp) + \beta p$$

$$= \alpha p + \beta p + \alpha(\frac{1}{2}Vp - \frac{1}{4}V)$$

$$= p(\alpha + \beta) + \alpha(\frac{1}{2}Vp - \frac{1}{4}V)$$

$$= p + \alpha\frac{1}{2}V(p - \frac{1}{2})$$

$$\Delta p = p - p'$$

$$= \alpha\frac{1}{2}V(p - \frac{1}{2})$$

With Assortative Mating:

- **$\Delta p \neq 0.0$ $\therefore$ change in allele frequency over time (i.e., evolution)**
- **if $p = 0.5$ the $\Delta p = 0.0$**
- **if $p < 0.5$ then $p => 0.0$**
- **if $p > 0.5$ then $p => 1.0$**
- **The frequency of assortative mating (α) only effects the RATE OF CHANGE not the stability or direction of change**

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